**MATHS SPECIALIST UNIT 4**

**INVESTIGATION 4**

**SIMPLE HARMONIC MOTION**

**No notes or calculators allowed in this validation.**

**Time Allowed: 50 minutes Total Marks: 49**

**Name: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

You should be familiar with the relationship between displacement, velocity and acceleration:

Displacement Velocity Acceleration

  

A special type of motion, called simple harmonic motion, occurs when an object oscillates on a straight line and at any time its acceleration is related to its position x by the differential equation:

, for some constant n and , where x is the distance of the body from the origin (x=0) and at points  the body is stationary (that is v=0).



**Part 1: Acceleration**

**1. [1, 2, 2 = 5 marks]**

Using the fact that , determine the magnitude and direction of the acceleration for the object at:

i) x = 0

ii) x = A

iii) x = -A

**2. [1, 1, 1 = 3 marks]**

Use the information from question 1 to complete the following:

i) The acceleration always points towards the ………………………………………………………..

ii) The magnitude of the acceleration is at a maximum at the ………………………………………

iii) The magnitude of the acceleration is at a minimum at the ………………………………………..

**Part 2: Velocity**

Using the differential equation , we can solve this to obtain an expression for the velocity of an object in simple harmonic motion.



**3. [6 marks]**

Complete the solution to the differential equation above to show that . (Remember that at points  the body is stationary, that is v=0)



**Part 3: Displacement**

Using the differential equation , we can solve this to obtain an expression for the

displacement of an object in simple harmonic motion.

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**4. [5 marks]**

Use the substitution  to solve the differential equation above to show the solution to the

integral is , where = constant of integration.

**5. [1 mark]**

The displacement , can be differentiated to give another expression for the velocity. Determine 

In conclusion for a body in simple harmonic motion we know:

i) ,

for some constant n and , where x is the distance of the body from the origin (x=0) and at points  the body is stationary (that is v=0).

ii) 



iii) 

Use these to answer the following questions:

**6. [3, 2, 2 = 7 marks]**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: , where x is the distance of the body from the origin x=0 and also the body is stationary at points .

1. Hence give an equation for the velocity of the body in terms of *x*, its position on the number line.
2. Given that: when t=0 the body is at x= +2.5 and the velocity is positive give the equation in the form  which shows the position x as a function time.
3. Alternatively: when t=0 the body is at x= +2.5 and the velocity is negative give the equation in the form  which shows the position x as a function time.

**7. [3, 7, 3 = 13 marks]**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: 

1. Hence give an equation for the velocity of the body in terms of *x*, its position on the number line.

(b) Use the substitution to show the solution to the integral

 is: 

(c) Given that: when t=0 the body is at x= +1.5 and the velocity is positive find the solution to the equation which shows the position x as a function time.

**8. [2, 2, 2, 3 = 9 marks]**

A body is moving with Simple harmonic motion on a straight line and the acceleration is given by the equation: where x is the distance of the body from the origin x=0 and also the body is stationary at points .

1. Using a change of origin where give the equation for the velocity of the body in the form 
2. Also express the velocity in the form 
3. From part (a) express the displacement equation in the form 
4. Also given that when t=0 the body is at x= +6.5 and the velocity is positive give the equation which shows the position x as a function time in the form:

